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Nonlinear Control of a Double Pendulum Electrodynamic Tether System

Hirohisa Kojima* and Tetsuro Sugimoto†
Tokyo Metropolitan University, Tokyo 191-0065, Japan

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Introduction

THE electrodynamic tether (EDT) system uses the Lorentz force as a thruster generated by the interference between the Earth's magnetic field and the electric current along the tether. The main advantage of this system is that its required electric current can be produced by solar power and electric particles such as Xenon, the mass of which is much smaller than those of traditional chemical propellants. As a result, this system is expected to be employed as a new space technology that will be useful in many types of space applications [1] such as reboosting of the international space station [2]; deorbiting of dysfunction satellites [3] (space debris removal system); the momentum exchange electrodynamic reboost (MXER) [4] system, which could be used to boost payloads from a low Earth orbit (LEO) to a geosynchronous transfer orbit (GTO); and scientific missions that could include the observation of meteors, the high-altitude atmosphere, and the aurora. The motion of the EDT system consists of librations that are similar to the motion of a pendulum and vibrations of the tether that are similar to the vibrations of a string. The librational motion of the tether system in an elliptic orbit is known to be chaotic [5,6]. The librational and vibrational motion of the tethered satellite system must be stabilized to successfully perform space operations such as observation of the atmospheric environment at high altitude. For the purpose of stabilization, a great number of control schemes have been proposed for various aspects of control [7], such as stationkeeping, deployment and retrieval control, generation of optimal trajectories [8], suppression of tether vibrations by electric current variation [9], periodic motion of the electrodynamic tether system on inclined orbits [10], and effect of the electrodynamic force on orbit [11]. However, to the best knowledge of the authors, except delayed feedback controls, application of nonlinear control schemes to the multipendulum type of electrodynamic tether systems for achieving constellation missions has not yet been widely studied.

The tether system treated in this Note consists of two subsatellites and a mother satellite, such as the space shuttle, connected in series via conductive massless tethers. Although this model seems simple, because the mass and flexibility of the tether are ignored, it can be used to investigate the behavior of the in-plane motion of an electrodynamic tether system in an elliptic orbit without significant computational effort and to investigate the applicability of the nonlinear controllers to the simple model.

In this study, two nonlinear control methods to stabilize the in-plane librational motion of an electrodynamic tether system in an elliptic orbit are investigated. The nonlinear controllers treated in this Note are a decoupling control method [6,12] and a model-following-decoupling control method [6,12]. The decoupling control can be used to control each tether attitude independently, and this independent motion is suitable for achieving satellite constellations. If this motion is realized for a three-mass tethered satellite system, various scientific missions will be possible, including the observation of planets with a magnetic field and the observation of the aurora from more than two directions simultaneously. The model-following-decoupling control method is introduced to achieve a periodic motion by which the Earth atmosphere at the specific altitude can be studied periodically. In this study, the periodic in-plane motion of a tethered satellite system in a circular orbit is employed as the reference trajectory for tracking by the actual tethered satellite system in an elliptic orbit.

The results of numerical simulations show that the nonlinear control schemes considered herein can stabilize the in-plane librational motion of a double pendulum electrodynamic tether system in an elliptic orbit.

Model Description

Electrodynamic Tether System

Although it is known that the instability of an electrodynamic tether system is associated with the out-of-plane motion, in this Note, to simplify the analysis, only in-plane motion is considered; gravity and the Lorentz force are treated as the only external forces affecting the tethered satellite system, and the center of mass of the system follows an elliptic orbit. A schematic model of the electrodynamic tether system treated in this Note is illustrated in Fig. 1. This system consists of a mother satellite, such as the space shuttle, two subsatellites, and two conductive tethers. The mother satellite and two subsatellites are labeled 0, 1, and 2, respectively, and are assumed as particle masses m_0, m_1, and m_2, respectively. Tether 1 and tether 2 are assumed to be conductive and rigid bodies without mass or inertia. Subsatellite 1 is connected to the mother satellite by tether 1 with length l_1 and is connected to subsatellite 2 by tether 2 with length l_2. Each satellite is assumed to be equipped with both anodic and cathodic contactors to generate both plus and minus electric current in tethers. This assumption may be an over-requirement for EDT systems, but is needed to implement the nonlinear control schemes concerned in this Note. The true anomaly of the center of mass of the system is denoted by the parameter η. The length of each tether is assumed to be 50 km, the mass of the mother satellite is assumed to be 100,000 kg, and the mass of each subsatellite is assumed to be 50 kg.

The definition of the reference frame for the system (x, y, z) is shown in Fig. 1. A dipole model is used to describe the Earth's magnetic field. Although the Earth's magnetic field must differ from the orbital axis of the tethered satellite system, in this study, for the purpose of simplifying the analyses, the magnetic field is assumed to be coincident with the orbital axis of the tether satellite, that is, it is given by

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*Associate Professor, Department of Aerospace Engineering; hkojima@cc.tmit.ac.jp. Member AIAA.

†Graduate Student, Department of Aerospace Engineering; u6891515@cc.tmit.ac.jp. Student Member AIAA.

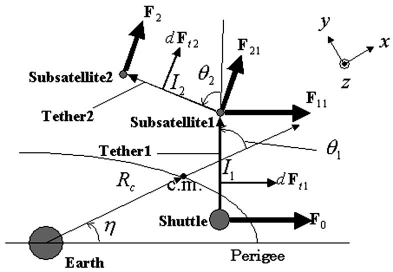

Fig. 1 Three-mass tether system model.

$$\boldsymbol{B} = \frac{\mu_m}{r^3}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{1}$$

where r is the distance from the center of the magnetic field to an arbitrary position and μ_m is the magnetic moment of the Earth's dipole (8.1×10^6 [T km^3]). The electrodynamic force induced in a conductive tether per unit length is given by

$$\mathrm{d}\boldsymbol{F}_{t1} = (I_1 \boldsymbol{l}_1 \times \boldsymbol{B})\,\mathrm{d}l_{t1}, \qquad \mathrm{d}\boldsymbol{F}_{t2} = (I_2 \boldsymbol{l}_2 \times \boldsymbol{B})\,\mathrm{d}l_{t2} \tag{2}$$

where $\boldsymbol{l}_1$ and $\boldsymbol{l}_2$ are unit vectors along tether 1 and tether 2, respectively; I_1 and I_2 are the currents in tether 1 and tether 2, respectively; and $\mathrm{d}l_{t1}$ and $\mathrm{d}l_{t2}$ are the unit lengths of tether 1 and tether 2, respectively.

To obtain simple equations of motion of this model, the Lorentz force affecting each tether is substituted by a force centralized to each satellite, as follows:

$$\boldsymbol{F}_0 = \int_0^{l_1/2} \mathrm{d}\boldsymbol{F}_{t1}, \qquad \boldsymbol{F}_{11} = \int_{l_1/2}^{l_1} \mathrm{d}\boldsymbol{F}_{t1}, \qquad \boldsymbol{F}_{21} = \int_0^{l_2/2} \mathrm{d}\boldsymbol{F}_{t2}$$
$$\boldsymbol{F}_2 = \int_{l_2/2}^{l_2} \mathrm{d}\boldsymbol{F}_{t2} \tag{3}$$

Note that in this Note, each substitutive force is assumed to be perpendicular to each corresponding tether, as shown in Fig. 1.

Equations of Motion

Considering the radius R_c, the true anomaly η, the angle of tether 1 θ_1, and the angle of tether 2 θ_2 as generalized coordinates, employing Kane's equation, the equations of motion are obtained in affine form as follows:

$$\boldsymbol{x} = [R_c, \eta, \theta_1, \theta_2, \dot{R}_c, \dot{\eta}, \dot{\theta}_1, \dot{\theta}_2]^T,$$
$$\frac{d^2}{\mathrm{d}t^2}\begin{bmatrix} R_c \\ \eta \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} f_1(\boldsymbol{x}) \\ f_2(\boldsymbol{x}) \\ f_3(\boldsymbol{x}) \\ f_4(\boldsymbol{x}) \end{bmatrix} + \begin{bmatrix} g_{11}(\boldsymbol{x}) & g_{12}(\boldsymbol{x}) \\ g_{21}(\boldsymbol{x}) & g_{22}(\boldsymbol{x}) \\ g_{31}(\boldsymbol{x}) & g_{32}(\boldsymbol{x}) \\ g_{41}(\boldsymbol{x}) & g_{42}(\boldsymbol{x}) \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{4}$$
$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \qquad \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where the electric currents I_1 and I_2 act as the control inputs, and the tether angles θ_1 and θ_2 are treated as the output variables. It should be noted that each input affects all of the general coordinates. A Mathematica (Wolfram Research, Inc.) code is implemented to create the equations of motion for the double pendulum electrodynamic tether system, by referring to Appendix A of [6].

Nonlinear Control of Librational Motion

Decoupling Control Method

A nonlinear differential equation in affine form with respect to the control input is given by

$$\dot{x} = f(x) + \sum_{i=1}^{s} g_i(x) u_i, \qquad \boldsymbol{x} \in \Re^n, \qquad \boldsymbol{y} = h(\boldsymbol{x}) \in \Re^p \tag{5}$$

where $\boldsymbol{x}$, $\boldsymbol{y}$, and u_i are the state vector, output vector, and control input, respectively. We consider the problem of finding the nonlinear control input by which each output can be controlled independently. This problem is called the Morgan problem, in which the dimension of the output p is equal to that of the control input s. The transformation to obtain such a nonlinear control input is referred to as the decoupling method. We apply this method to control the tether angles of the EDT system, θ_1 and θ_2, independently.

The nonlinear control input to decouple the system is given by

$$\boldsymbol{u} = b^{-1}(\boldsymbol{x})[-a(\boldsymbol{x}) + \boldsymbol{v}] \qquad a(\boldsymbol{x}) = \begin{bmatrix} L_f^{r_1} h_1(\boldsymbol{x}) \\ \cdots \\ L_f^{r_s} h_s(\boldsymbol{x}) \end{bmatrix},$$
$$b(\boldsymbol{x}) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(\boldsymbol{x}) & \cdots & L_{g_s} L_f^{r_1-1} h_1(\boldsymbol{x}) \\ & \vdots & \\ L_{g_1} L_f^{r_s-1} h_s(\boldsymbol{x}) & \cdots & L_{g_s} L_f^{r_s-1} h_s(\boldsymbol{x}) \end{bmatrix} \tag{6}$$

where v is the fictitious control input, r_k $(k = 1, \cdots, s)$ is the relative degree, and $L_f h(\boldsymbol{x})$ is the Lie derivative of $h(\boldsymbol{x})$ along f.

The relative degrees r_1 and r_2 of the system with respect to the outputs h_1 and h_2, respectively, are obtained as $r_1 = 2$ and $r_2 = 2$, by using a Mathematica code given in Appendix B of [6]. Thus, we have

$$a(\boldsymbol{x}) = \begin{bmatrix} f_3(\boldsymbol{x}) \\ f_4(\boldsymbol{x}) \end{bmatrix}, \qquad b(\boldsymbol{x}) = \begin{bmatrix} g_{31}(\boldsymbol{x}) & g_{32}(\boldsymbol{x}) \\ g_{41}(\boldsymbol{x}) & g_{42}(\boldsymbol{x}) \end{bmatrix} \tag{7}$$

Employing the feedback control

$$\begin{bmatrix} u_1 & u_2 \end{bmatrix}^T = b^{-1}(\boldsymbol{x})[-a(\boldsymbol{x}) + \boldsymbol{v}] \tag{8}$$

yields the decoupled system

$$\begin{bmatrix} \ddot{h}_1 \\ \ddot{h}_2 \end{bmatrix} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{9}$$

To achieve the reference angles in which both of the subsatellites point to the Earth, the following proportional and derivative (PD) type of feedback control is chosen to generate the fictitious control inputs in Eq. (9), v_1 and v_2, as follows:

$$v_1 = -K_{P1}(\theta_1 - \pi) - K_{D1}\dot{\theta}_1, \qquad v_2 = -K_{P2}\theta_2 - K_{D2}\dot{\theta}_2 \tag{10}$$

where K_{P1}, K_{D1}, K_{P2}, and K_{D2} are constant control gains.

Model-Following-Decoupling Control Method

The model-following-decoupling control method is initially used to specify the reference model that behaves desirably and is later used to control the response of a plant to converge asymptotically to that of the reference model by providing feedback control inputs based on the difference between the responses.

Consider the plant model and the reference model described by the affine differential equation:

$$\dot{\boldsymbol{x}}_P = f_P(\boldsymbol{x}_P) + g_P(\boldsymbol{x}_P)\boldsymbol{u}_P, \qquad \boldsymbol{x}_P \in \Re^{n_P}, \qquad \boldsymbol{u}_P \in \Re^{s_P},$$
$$\boldsymbol{y}_P = h_P(\boldsymbol{x}_P) \in \Re^{p_P} \tag{11}$$

$$\dot{\boldsymbol{x}}_M = f_M(\boldsymbol{x}_M) + g_M(\boldsymbol{x}_M)\boldsymbol{u}_M, \qquad \boldsymbol{x}_M \in \Re^{n_M}, \qquad \boldsymbol{u}_M \in \Re^{s_M},$$
$$\boldsymbol{y}_M = h_M(\boldsymbol{x}_M) \in \Re^{p_M} \tag{12}$$

The difference between the output of the reference model and that of the plant model

$$h(\boldsymbol{x}) = \boldsymbol{e} = h_M(\boldsymbol{x}_M) - h_P(\boldsymbol{x}_P) \tag{13}$$

is treated as the output of the extended system that combines the plant model and the reference model.

Let us consider the case in which the plant is controlled by

$$\boldsymbol{u}_P = \alpha(\boldsymbol{x}) + \beta(\boldsymbol{x})\boldsymbol{u}_M \tag{14}$$

and the reference model is controlled by an arbitrary control input $\boldsymbol{u}_M$. In this case, if the error e asymptotically converges to zero as time increases, then the plant can be regarded as following the reference model.

The tethered satellite system in an elliptic orbit is treated as the controlled plant model. Although the tethered satellite system in an elliptic orbit can be used as the reference model when the motion of the system has been changed to an adequate degree to periodic motion by control, to simplify the problem, the formulation is shown only for the case in which the reference model in a circular orbit is employed.

The state vector, control inputs, and outputs of the plant model are given by

$$\boldsymbol{x}_P = [x_{P1}, \ldots, x_{P8}]^T = [R_c, \eta, \theta_{P1}, \theta_{P2}, \dot{R}_c, \dot{\eta}, \dot{\theta}_{P1}, \dot{\theta}_{P2}]^T \in \Re^8 \tag{15}$$

$$\boldsymbol{u}_P = [u_{P1}, u_{P2}]^T = [I_{P1}, I_{P2}]^T \in \Re^2 \tag{16}$$

$$\boldsymbol{y}_P = [y_{P1}, y_{P2}]^T = [\theta_{P1}, \theta_{P2}]^T = [x_{P3}, x_{P4}]^T \in \Re^2 \tag{17}$$

Similarly, the state vector, control inputs, and outputs of the reference model for the tethered satellite system in a circular orbit are given by

$$\boldsymbol{x}_M = [x_{M1}, \ldots, x_{M4}]^T = [\theta_{M1}, \theta_{M2}, \dot{\theta}_{M1}, \dot{\theta}_{M2}]^T \in \Re^4 \tag{18}$$

$$\boldsymbol{u}_M = [u_{M1}, u_{M2}]^T = [I_{M1}, I_{M2}]^T \in \Re^2 \tag{19}$$

$$\boldsymbol{y}_M = [y_{M1}, y_{M2}]^T = [\theta_{M1}, \theta_{M2}]^T = [x_{M1}, x_{M2}]^T \in \Re^2 \tag{20}$$

In this case, the relative degree of the plant models γ_1 and γ_2 and the relative degree of the reference models π_1 and π_2 are obtained as $\gamma_1 = \gamma_2 = 2$ and $\pi_1 = \pi_2 = 2$, respectively. The matrices $Q_P(\boldsymbol{x}_P)$ and $Q_M(\boldsymbol{x}_M)$ for the preceding models are obtained, respectively, as follows:

$$Q_p(\boldsymbol{x}_P) = \begin{bmatrix} L_{g_{P1}} L_{f_P}^1 h_{P1} & L_{g_{P2}} L_{f_P}^1 h_{P1} \\ L_{g_{P1}} L_{f_P}^1 h_{P2} & L_{g_{P2}} L_{f_P}^1 h_{P2} \end{bmatrix} \tag{21}$$

$$Q_M(\boldsymbol{x}_M) = \begin{bmatrix} L_{g_{M1}} L_{f_M}^1 h_{M1} & L_{g_{M2}} L_{f_M}^1 h_{M1} \\ L_{g_{M1}} L_{f_M}^1 h_{M2} & L_{g_{M2}} L_{f_M}^1 h_{M2} \end{bmatrix} \tag{22}$$

The matrix $Q_P(\boldsymbol{x}_P)$ is a full-rank matrix. This means that the necessary and sufficient condition for the existence of the control law is satisfied. Therefore, the functions $\alpha(\boldsymbol{x})$ and $\beta(\boldsymbol{x})$ for the model-following-decoupling control law can be obtained as

$$\alpha(\boldsymbol{x}) = Q_P(\boldsymbol{x}_P)^{-1} \left[\begin{bmatrix} L_{f_M}^2 h_{M1} \\ L_{f_M}^2 h_{M2} \end{bmatrix} - \begin{bmatrix} L_{f_P}^2 h_{P1} \\ L_{f_P}^2 h_{P2} \end{bmatrix} + z \right] \tag{23}$$

$$\beta(\boldsymbol{x}) = Q_P(\boldsymbol{x}_P)^{-1} Q_M(\boldsymbol{x}_M) \tag{24}$$

where $\boldsymbol{z}$ is a fictitious control input vector with s dimension to specify the nature of the model-following decoupled system. Because the relative degrees γ_1, γ_2, π_1, and π_2 are equal to two, the following PD feedback control can be employed to generate the fictitious control input vector $\boldsymbol{z}$:

$$z_1 = K_{P1} e_1 + K_{D1} \dot{e}_1, \qquad z_2 = K_{P2} e_2 + K_{D2} \dot{e}_2 \tag{25}$$

where K_{P1}, K_{D1}, K_{P2}, and K_{D2} are constant control gains. The model-following decoupled system resulting from the preceding nonlinear feedback control is obtained as

$$\ddot{e}_1 = -z_1 = -K_{P1} e_1 - K_{D1} \dot{e}_1, \qquad \ddot{e}_2 = -z_2 = -K_{P2} e_2 - K_{D2} \dot{e}_2 \tag{26}$$

The poles of the decoupled system can be chosen arbitrarily by setting the preceding control gains.

Numerical Simulation

Results of the Decoupling Control Method

The initial states of the system are given as follows: $R_c = 6600$ [km], $\eta = 0$ [rad], $\theta_1 = \pi$ [rad], $\theta_2 = 0.2$ [rad], $\dot{R}_c = 0$ [km/s], $\dot{\eta} = 1.28994 \times 10^{-3}$ [rad/s], $\dot{\theta}_1 = 0$ [rad/s], and $\dot{\theta}_2 = 0$ [rad/s]. The orbital period τ is determined to be 7457 s, and the eccentricity of the mass center of the system is found to be 0.2. The fictitious control inputs v_1 and v_2 are calculated by Eq. (10), in which the control gains for the decoupling control are set as $K_{P1} = K_{P2} = 2.5 \times 10^{-6}$ [A/rad] and $K_{D1} = K_{D2} = 1.0 \times 10^{-3}$ [As/rad]. These control gains are selected so that the magnitude of the electric current does not considerably exceed 10 A, which is assumed to flow on the conductive tethers, in this Note. The time responses of tether angles θ_1 and θ_2 and the magnitude of the currents I_1 and I_2 are shown in Figs. 2a–2c, respectively. Figures 2a and 2b show that the tether angle θ_2 is successfully controlled by the decoupling method without any changes in the tether angle θ_1. This result indicates the validity of the decoupling method presented herein. Figure 2c shows that the currents in tether 1 and tether 2 oscillate even after the tether angles converge to zero. This is because the present electrodynamic tether system is in an elliptic orbit.

Results of the Model-Following-Decoupling Control Method

The tethered satellite system in a circular orbit with the radius of 8250 km is treated as the reference model. The orbital period of this reference model is 7457 s. The plant model is assumed to be a tethered satellite system in an elliptic orbit with an eccentricity of 0.2 and a radius of perigee of 6600 km. The orbital period of this plant at the initial time is the same as that of the reference model (7457 s). The reference system is given by $\theta_{M1} = \pi + 0.1$ [rad], $\theta_{M2} = 0$ [rad], $\dot{\theta}_{M1} = 0$ [rad/s], and $\dot{\theta}_{M2} = 0$ [rad/s] at the initial time, and no control is used to actuate the reference system, that is, $\boldsymbol{u}_M = [0, 0]^T$. The plant is given the initial conditions of $R_c = 6600$ [km], $\eta = 0$ [rad], $\theta_{P1} = 0.9\pi$ [rad], $\theta_{P2} = 0.2$ [rad], $\dot{R}_c = 0$ [km/s], $\dot{\eta} = 1.28994 \times 10^{-3}$ [rad/s], $\dot{\theta}_{P1} = 0$ [rad/s], and $\dot{\theta}_{P2} = 0$ [rad/s].

The control gains are set as $K_{P1} = K_{P2} = 1.0 \times 10^{-6}$ [A/rad] and $K_{D1} = K_{D2} = 1.0 \times 10^{-3}$ [As/rad] in the present simulation. When these control gains are used, the eigenvalues of the decoupled system are obtained as $10^{-3} \times (0.5 \pm j\sqrt{3}/2)$. Namely, the settling time of the tether angles is estimated as 8000 s, that is, approximately one orbital period. If the control gains are selected so that the electric current becomes smaller, then the settling time of the system becomes longer than the preceding settling time.

The oscillations of the tether angles of the reference model are approximately sinusoidal waves, as shown in Figs. 3a and 3b. The time response of the plant controlled to follow the motion of the reference model are also shown in Figs. 3a and 3b. These figures show that the response of the plant model successfully follows that of the reference model and that the errors converge to zero within approximately one orbital period. The time required for the convergence of the errors indicates the validity of the model-following-decoupling control method presented herein. The time histories of the control inputs are shown in Fig. 3c, which shows that the amplitudes of the currents are almost the same as those of the previous simulation result. This may be because the periodic motion of the reference model is not near the desired motion for the plant model, which needs no control to keep the periodic motion.

Finally, it is interesting to see in Figs. 2c and 3c that the sign of the electric currents in tethers 1 and 2 is almost always opposite. In other words, the Lorentz force along one tether works as a booster for the system, whereas the other one works as a deorbiter. This may be because the resulting total Lorentz torque around the center of mass

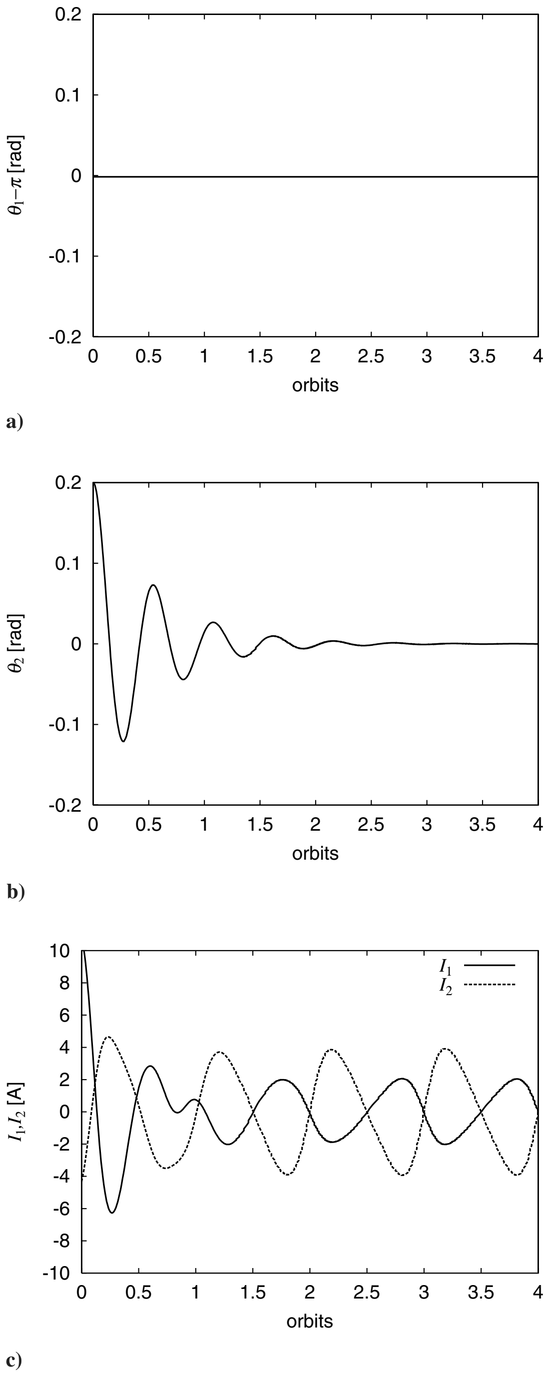

Fig. 2 Time responses of a) tether angle 1, b) tether angle 2, and c) time histories of electric currents resulting from the decoupling control method.

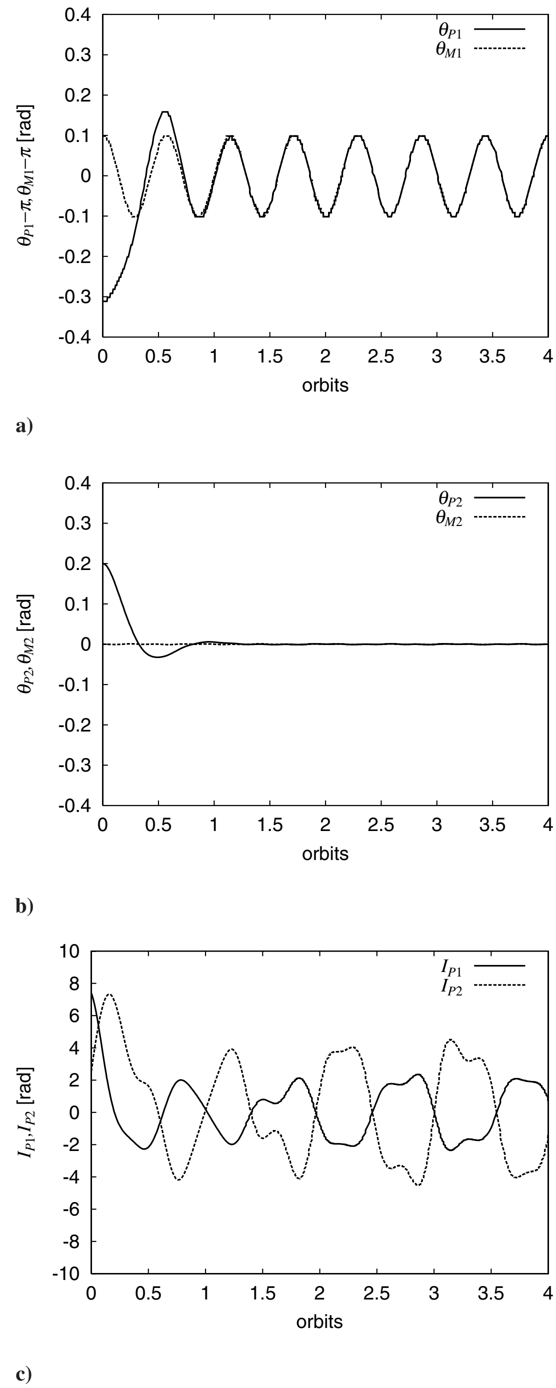

Fig. 3 Time responses of a) tether angle 1, b) tether angle 2, and c) time histories of electric currents resulting from the model-following-decoupling control method.

of the system is effective to control the in-plane librational motion of the tether system.

Conclusions

In this Note, two nonlinear controllers (i.e., a decoupling control method and a model-following-decoupling control method) were investigated with respect to the stabilization of the chaotic, librational motion of electrodynamic tethers by changing the current along the tethers. Numerical simulations revealed that the chaotic, librational motion of each angle of the electrodynamic tethers can be stabilized independently using the decoupling control method. The model-following-decoupling control method was applied to the electrodynamic tethered system so that a tethered satellite system in an elliptic orbit was able to track the periodic motion of the reference system in a circular orbit. The numerical simulation of the model-following-decoupling control method revealed that the control method has good performance for tracking the trajectory of the reference system.

Future work includes study on the applicability of the concerned controllers to more realistic models, including mass and flexibility of tethers and plasma density depending on the orbital altitude.

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D. Spencer
Associate Editor